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Error modeling and improved position estimation for optical incremental encoders by means of time stamping

Roel Merry, René van de Molengraft and Maarten Steinbuch

Abstract— **Optical incremental encoders are extensively used for position measurements in motion systems. The number of slits on the encoder disk defines the resolution of the encoder and bounds the accuracy of the position measurement. The encoder position measurements suffer from quantization errors. Moreover, encoder imperfections occur due to manufacturing tolerances. In this paper a method, which is based on time stamping, is proposed to obtain more accurate position and velocity estimations. Time stamping makes use of stored events, consisting of the encoder counts and their time instants, which are captured at a high resolution clock. Low order polynomial fitting through these encoder events is combined with active compensation of the encoder errors. The proposed method is applied in real-time experiments to a motion system. The results show an improvement in the position accuracy of up to 87% in terms of the maximum error. The estimated velocity is also much more accurate than the differentiated quantized encoder output signal.**

I. INTRODUCTION

Optical incremental encoders are commonly used for position measurements in motion control systems and are available in a rotational and linear form. They consists of three basic components: a slotted disk, a light source and a dual light detector. The light source shines on the disk which has a regularly spaced radial pattern of transmissive and reflective elements, called encoder increments. The dual light detector detects the amount of light passed through the slotted disc. The outputs of the light detector are used by a signal processing circuitry, which produces two output signals that have a relative phase difference of 90°. The direction of motion can be determined by detecting the leading signal. In digital encoders, the two detector outputs are converted into digital pulse quadrature signals using thresholding circuits. A digital counter counts the up and down changes of the pulse signals and uses the counter value as an estimation for the encoder position. The resolution of the counter is limited by the number of slits on the encoder disk.

Optical incremental encoders are widely used to apply feedback control on motion systems where the position is measured at a fixed sampling frequency. The position accuracy is limited by the quantized position measurement of the encoder. The velocity information is often obtained by numerical differentiation of the quantized position signal. The limited resolution and quantization error limit the performance in high accuracy control applications. Furthermore the velocity acquired by differentiation shows large spikes due to the quantization. More accurate position and velocity information can be obtained by using expensive encoders with more increments or by smart signal processing techniques.

In literature several methods have been proposed to improve the position and/or the velocity estimations. The methods can be divided into two kinds; fixed-time methods, which measure the counter value over a fixed time period, and fixed-position methods, which measure the time necessary to travel over a fixed amount of counts. For real-time control purposes a fixed-time method is desired since the controller is evaluated at fixed time intervals. A hybrid algorithm which switches between fixed-time and fixed-position measurements as proposed in [1] is therefore not suitable for control applications. Also algorithms which adjust the sampling interval based on the momentary velocity estimation [2] or which switch between sensors with different sampling frequencies based on a velocity threshold [3] are not applicable in real control.

Fixed-time methods are developed that either require a model, such as the Kalman based methods [4], [5], [6], or that use only the measured quadrature counts [7], [8], [9], [10], [11]. Also methods are proposed that incorporate encoder errors. The encoder errors are regarded to have a stochastic nature [12], are assumed negligible small [3], or are assumed to be repetitive over a small number of increments [13]. In [14], a lookup table describing the encoder errors is derived using neural networks.

In this paper, a fixed-time method is proposed in which the angular position and velocity are estimated by extrapolating a polynomial fit through a number of quadrature counts and their corresponding time instants, called encoder events. The proposed method makes it possible to estimate a more accurate velocity than methods which estimate the velocity using only the quadrature counts. The method is fully data based, which has an advantage that no model is required. Furthermore, in this paper modeling of the encoder errors shows that the encoder errors are highly repetitive over one revolution, which allows the encoder errors to be compensated for by a calibration of the encoder.

The main contributions of this paper are threefold. Firstly, position and velocity estimations are obtained by a least squares polynomial fit through a number of encoder events, which are measured at a high resolution clock, instead of using the counter values at the controller sampling intervals. Secondly, different kinds of encoder imperfections, such as eccentricity and non-equidistant slit positions, are modeled and their effects are analyzed by means of simulations. In

R.J.E. Merry, M.J.G. van de Molengraft and M. Steinbuch are with the Control Systems Technology group, Department of Mechanical Engineering, Technische Universiteit Eindhoven, 5600 MB Eindhoven, The Netherlands, r.j.e.merry@tue.nl, http://www.dct.tue.nl

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the experiments the encoder imperfections are accounted for by means of calibration. Finally, since the proposed method is ultimately of the fixed-time kind, it is applicable for real-time experiments.

This paper is organized as follows. The time stamping concept is explained in more detail in Section II. Different kinds of encoder imperfections and their effect on the position measurement are discussed in Section III. The position reconstruction method is described in Section IV. In Section V, the efficiency of the proposed method and the effects of the encoder imperfections are shown by means of simulations. Experimental results on a motion system are presented in Section VI. Finally, conclusions will be drawn in Section VII.

II. Time stamping concept

In most motion control applications the position is measured by reading out the encoder counter value at the fixed sampling rate of the controller. This introduces even for ideal encoders a quantization error in the position measurement of maximally half an encoder count.

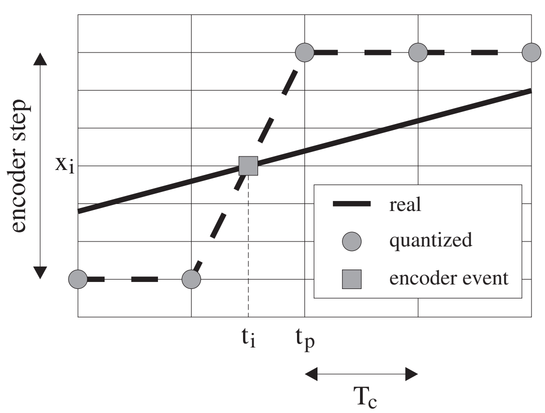

Fig. 1. Concepts of time stamping

A possibility for increasing the accuracy of the position measurement with the same resolution encoders is using the concept of time stamping [15]. The quantized signal contains the encoder counter value at the sample times of the controller t_p, as can be seen in Fig. 1. The counter values are read at a fixed sampling rate T_c. The time stamping concept stores the time instants t_i of a number of encoder transitions together with their position x_i. The pair (t_i, x_i) is called an encoder event.

If encoder events are used for position or velocity feedback control, a fixed sampling frequency is not straightforward since the encoder events are obtained at a variable frequency proportional to the instantaneous velocity of the system during the measurement. To obtain a position or velocity estimation at the equidistant sample times of the controller, a function is fitted through a number of past encoder events. This function is extrapolated to the desired time instant of the controller.

The time stamps also contain information about the encoder imperfections. The imperfections can be identified by driving the encoder at a constant velocity. The identified footprint of the encoder errors can be used to adjust the encoder events according to their index number. Calibration of the encoder errors makes it possible to compensate for the imperfections.

III. Encoder imperfections

If optical encoders are considered as stand-alone mechatronic systems, they incorporate both mechanical and electrical errors in their output signals such as

- The non-uniform encoder slit distribution on the encoder disc due to course manufacturing, damage or blocking by dust particles.
- Misplacement of the sensing photodiodes and the light sources.
- Eccentricity or tilt of the encoder disc due to the manufacturing or due to incorrect mounting of the disc on the rotating shaft.
- Errors caused by the electronic circuitry.

The above imperfections mainly lead to a deterministic reproducible deviation of the position measurement. Most of the information available on the type of errors and their sources can be derived from the data sheet of the encoders.

The encoder errors can be specified as phase shifts, in electrical degrees °e, of the rising or falling edges of the quadrature pulses of the encoder output signal. The errors can be regarded as a percentage of the cycle C, defined as the amount of rotation between two rising edges of channel A. One complete cycle corresponds to 360°e. The main error sources are the following

Cycle error
: Indicates the cycle uniformity; the difference between an observed shaft angle which gives rise to one electrical cycle and the nominal angular increment. The cycle error causes the rising edge of the time stamps of channel A to be shifted.

Pulse width error
: The deviation of the pulse width from its ideal value of 180°e. The pulse width error introduces a time shift between the time stamps of the rising edge and falling edge of an equal channel.

Phase error
: The deviation of the phase between channel A and B from its ideal value of 90°e caused by the misalignment of the two light sensors. The phase error introduces a time shift between the time stamps of channel A with respect to the edges of channel B.

Eccentricity
: Eccentricity of the code-wheel introduces a time shift to all edges. The amount of shifting describes a sinusoid with a period of one revolution.

IV. Position reconstruction

For the position and velocity estimations, a low order polynomial is fitted by the least squares method through a number of encoder events. Let n be the number of encoder events, m the order of the polynomial fit, $p_{0,\ldots,m}$ the number of polynomial coefficients to be estimated, $t_{1,\ldots,n}$ the time information of the encoder events, and $x_{1,\ldots,n}$ the position information of the encoder events. Now define

$$A = \begin{bmatrix} t_1^m & t_1^{m-1} & \cdots & 1 \\ t_2^m & t_2^{m-1} & \cdots & 1 \\ \vdots & \vdots & \cdots & 1 \\ t_3^m & t_3^{m-1} & \cdots & 1 \end{bmatrix}, \quad P = \begin{bmatrix} p_m \\ p_{m-1} \\ \vdots \\ p_0 \end{bmatrix}, \quad B = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}.$$

If $n = m$, an exact fit is made through the events, for the least squares method $n > m$. The over-determined system of linear equations to be solved for the polynomial fit equals

$$AP = B.$$

The least squares method for calculating the polynomial coefficients can be formalized as

$$A^T AP = A^T B. \tag{1}$$

Since the polynomial coefficients P of (1) have to be calculated in real-time, LU-factorization without pivoting is used. Since the position and velocity estimations are required at the sampling times of the controller, the fitted polynomial with coefficients P is extrapolated to the desired time instant. The extrapolation of the polynomial to the time instant t_e results in an estimated position $\tilde{x}_e$ and estimated velocity $\dot{\tilde{x}}_e$ as

$$\tilde{x}_e|_{t=t_e} = p_m t_e^m + p_{m-1} t_e^{m-1} + \ldots + p_0,$$
$$\dot{\tilde{x}}_e|_{t=t_e} = m p_m t_e^{m-1} + (m-1) p_{m-1} t_e^{m-2} + \ldots + p_1.$$

V. SIMULATIONS

To evaluate the benefits of time stamping and the polynomial fitting method, simulations are performed. Also the effect of the encoder error sources is analyzed by means of simulations. Fig. 2 shows the simulation scheme, where the encoder is simulated by a quantization of the generated true position signal. The encoder events are stored in a register which can hold up to five encoder events. The size of the register is chosen such that it matches the size of the register of the used data acquisition device for the experiments (see Section VI). The effects of the different encoder error sources are implemented as perturbations of the time stamps of the encoder events. The encoder events are used as input for the polynomial fitting procedures of different orders. Finally, the estimated positions and velocities are obtained by extrapolation of the fitted polynomials.

Fig. 3 shows the position and error signals for a simulation with a constant velocity of 400 counts/s, which is equal to a rotational speed of 60 rpm for an encoder with 100 slits per revolution. The quantization of the encoder signal can be seen by the stair-case shaped signal with a step size of one increment, leading to an error of ±0.5 counts. The time stamping concept leads for a first, second, and third order fit to a position signal which is equal to the reference signal and a zero position error. Note that this is without encoder imperfections.

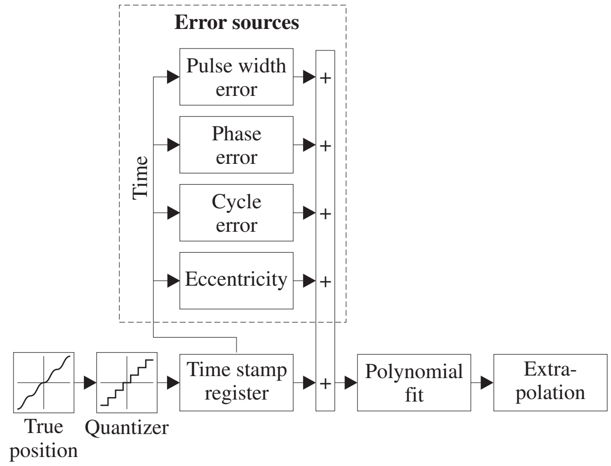

Fig. 2. Simulation scheme

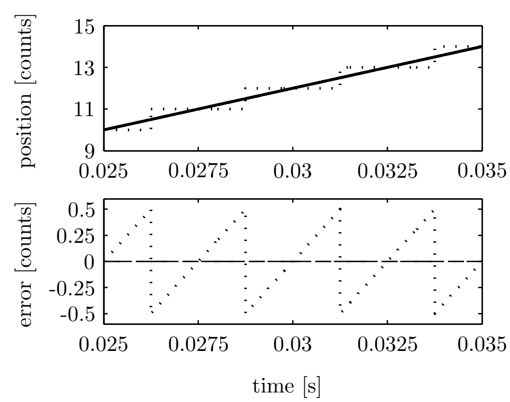

Fig. 3. Positions and errors of the quantized signal (dotted), first order fit (dash dotted), second order fit (dashed) and third order fit (grey dashed) for a constant velocity reference signal of 400 counts/s (solid) without encoder imperfections

Suppose that the true position signal will not exactly follow a constant velocity, but is perturbed by a sinusoidal disturbance of 10% of the nominal speed as

$$x_{\text{true}} = vt + (0.1/2\pi f_{\text{dist}} v \sin(2\pi f_{\text{dist}} t), \tag{2}$$

where the disturbance frequency $f_{\text{dist}} = 100$ Hz and the velocity $v = 400$ counts/s. The position errors of the simulations with (2) are shown in Fig. 4. The quantized signal again has an error of ±0.5 counts. The first order fit results in a smaller error of $5.5 \cdot 10^{-2}$ counts, but is not able to follow the disturbance on the input signal. The second and third order fits result in errors of respectively $4.8 \cdot 10^{-3}$ counts and $7.8 \cdot 10^{-4}$ counts. In Fig. 4, the resulting errors of the second order fit (dashed) and the third order fit (grey dashed) coincide.

For the simulations with encoder imperfections, the data of the HEDS-5540 encoder, which is used in the experiments, is used (see Table I)[16]. Fig. 5 depicts the position error of a simulation with the reference signal of (2) and the encoder imperfections of Table I. No fit order is able to compensate for the encoder imperfections. However, the errors are still smaller than for the quantized signal. Since the encoder

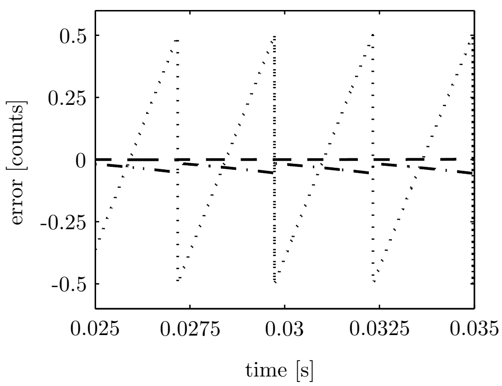

Fig. 4. Errors of the quantized signal (dotted), first order fit (dash dotted), second order fit (dashed) and third order fit (grey dashed) for a constant velocity reference signal with a 10% velocity perturbation without encoder imperfections

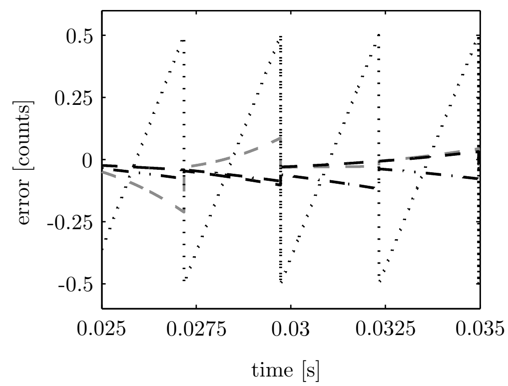

Fig. 5. Errors of the quantized signal (dotted), first order fit (dash dotted), second order fit (dashed) and third order fit (grey dashed) for a constant velocity reference signal with a 10% velocity disturbance and encoder imperfections

TABLE I

ERRORS FOR THE HEDS-5540 ENCODER

Description	Error [°e] Typical	Maximum
Cycle error	3	5.5
Pulse width error	5	35
Phase error	2	15
Eccentricity	0.04 mm	
Electrical characteristics at 25°C	Rise time: 180 ns Fall time: 40 ns	

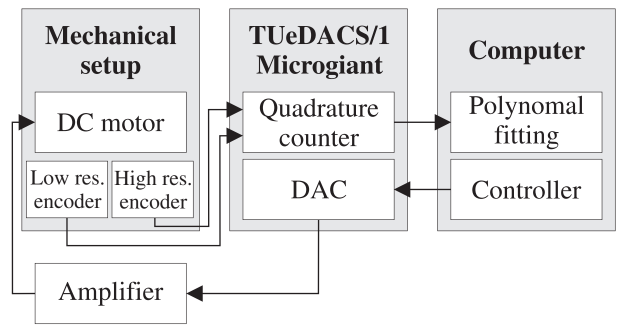

Fig. 6. Block diagram of the experimental setup

errors are highly reproducible and repeat with a period of one revolution, they can be actively compensated for in the position estimation.

The simulations show that the time stamping concept significantly improves the position estimation. For the experiments, a second order fit will be used since it has a much smaller error compared to the first order fit and is computationally cheaper than the third order fit, which is important for a real-time implementation.

VI. EXPERIMENTS

In this section, the results of the experiments, where the time stamping concept was applied to a motion system, are discussed. Experiments are performed at a constant reference velocity and with a second order polynomial fit.

A. Experimental setup

The experimental setup consists of the mechanical setup, an amplifier, a TUeDACs Microgiant data acquisition device [17] and a computer as shown in the block diagram of Fig. 6.

The mechanical setup, shown in Fig. 7, consists of a DC motor that is connected to a rotating mass. On the DC motor, a HEDS-5540 encoder [16] with a resolution of 100 slits/revolution is mounted. On the opposite site, a Heidenhain ROD-426 encoder with 5000 slits/revolution is connected to the rotating mass. The output of the Heidenhain encoder will be used as a reference to determine the improvement of the time stamping concept.

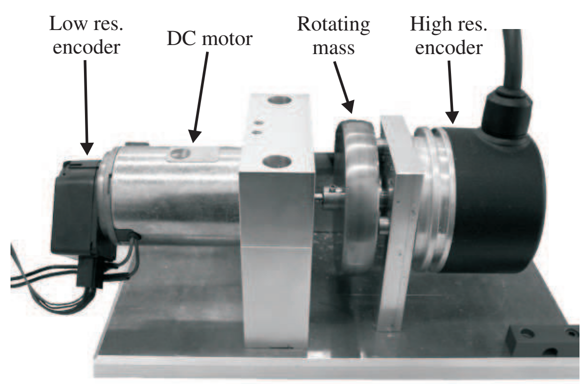

Fig. 7. The mechanical setup

The data acquisition device, shown in Fig. 8, is equipped with 32 bit quadrature counters with a maximum count frequency of 10 MHz and can generate encoder events with a time resolution of 50 ns [17]. The five most recent encoder events are stored in a register. Furthermore, the Microgiant is equipped with a DAC output, which is used to drive the mechanical setup.

Xenomai Linux and a real-time USB stack are used for the

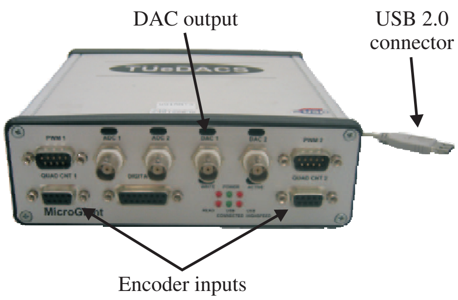

Fig. 8. The TUeDACs Microgiant data acquisition device

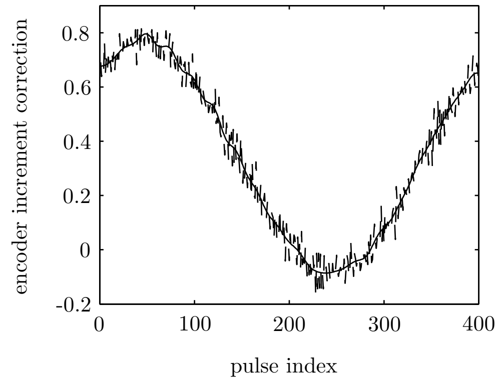

Fig. 9. Look up table encoder imperfections (dashed) and a low-pass filtered version (solid)

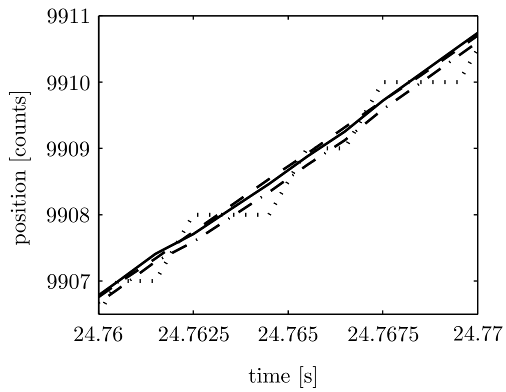

Fig. 10. Position signals of the high resolution encoder (dashed), low resolution encoder (dotted), time stamping without lookup table (dash-dotted) and time stamping with lookup table (solid)

real-time communication, at a fixed sampling rate of 1 kHz, between the computer and the Microgiant. The computer reads the time stamping registers of the Microgiant for the polynomial fitting and generates the control signal to the system in order to maintain a constant rotating velocity.

B. Results

To follow the constant velocity reference profile, the system is feedback controlled. Using a measured frequency response function (FRF) of the system from the motor input to the position of the high resolution encoder, a feedback controller is designed, which results in a bandwidth of the controlled system of 10 Hz.

Since most encoder error sources are very reproducible, the low resolution encoder can be calibrated. For the calibration, the high resolution encoder is used as a reference. The reference velocity for the calibration equals 400 counts/s. The first half of the data is used to construct a lookup table representing the encoder errors. The second half of the data will be used as validation data. With the data, a lookup table is constructed by matching each increment of the low resolution encoder with the output of the high resolution encoder and averaging over a number of revolutions. The lookup table, depicted in Fig. 9, can be used to adjust the encoder events according to the pulse index of the encoder. The lookup table shows that the encoder imperfections have a low frequent harmonic part with a period of one revolution. This error is probably caused by a combination of the eccentricity and tilt of the code-wheel. Other error sources, such as the non-equidistant distribution of the slits on the code-wheel, are also present in the lookup table.

The positions of the low and high resolution encoders for a reference velocity of 400 count/s are shown in Fig. 10. The stair-case signal of the low resolution encoder can be clearly seen. The time stamping concept is applied using the last five encoder events of the Microgiant and a second order fit. The lookup table is used to adjust the encoder events according to their index number. The positions obtained by applying time stamping with and without the lookup table are also shown in Fig. 10. With time stamping, the position signal is much smoother than the position signal of the low resolution encoder signal. However, the position signal obtained with time stamping has an offset compared to the reference signal. Application of the lookup table significantly reduces the effect of encoder imperfections and thus the offset. The resulting position resembles the reference signal much better.

Fig. 11 shows the position errors of the low resolution encoder and of the estimated positions using the time stamping concept with and without lookup table. The error of the low resolution encoder equals maximally 1.32 counts and clearly shows a sinusoidal shape due to the encoder imperfections. Furthermore, the quantization of the output of the low resolution encoder error can be seen by the band of ± 0.5 counts in the sinusoidal shaped error. With time stamping, the maximum error reduces by 33% to 0.88 counts and the band reduces to ± 0.2 counts. Finally, adjusting the encoder events with the lookup table removes the sinusoidal part as can be seen in Fig. 11. The resulting maximum error equals 0.17 counts, which is a reduction of 87%.

The velocity estimation of the proposed method is much smoother than the differentiated quadrature signal of the low resolution encoder, as depicted in Fig. 12.

VII. Conclusions

Position measurements obtained from optical encoders suffer from quantization errors of maximally half an encoder count and from several kinds of encoder imperfections, which disturb the position measurement. The proposed method makes use of time stamping. A polynomial

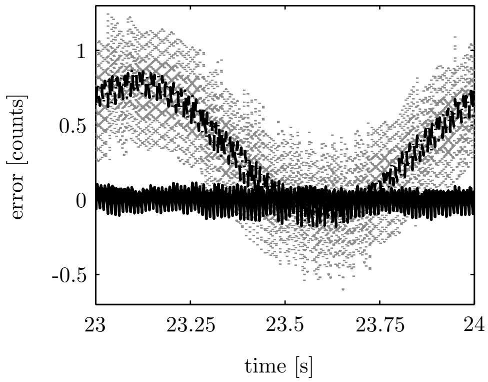

Fig. 11. Position errors of low resolution encoder (grey, dotted), time stamping without lookup table (dash-dotted) and time stamping with lookup table (solid)

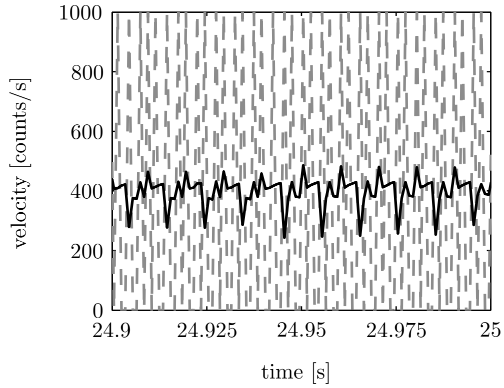

Fig. 12. Velocity obtained by differentiation of the low resolution quadrature signal (grey dashed) and the velocity estimation of the time stamping method (solid)

function is fitted through encoder events, consisting of the counter value and their time instants. The fitted polynomial is extrapolated to the desired time instant. In the proposed method, also encoder imperfections incorporated. The estimated encoder position with the proposed method is much more accurate than the quantized encoder output.

The encoder imperfections from different sources are identified separately. The most evident error sources are the eccentricity of the code-wheel and the misalignment of the LED-sensor combination. These error sources are both dependent on the rotational speed.

The simulations show the effectiveness of the time stamping concept combined with polynomial fitting and extrapolation. The second order fit provides a good tradeoff between accuracy and computational costs. The encoder imperfections are not accounted for by the polynomial fits. However, the encoder imperfections are highly reproducible and are compensated separately.

The incorporation of the time stamping concept in the TUeDACs Microgiant data acquisition device makes it possible to apply the proposed method in real-time experiments.

In practice, the encoder imperfections can be identified using an accurate reference, which is in this paper a second encoder with a much higher resolution. A lookup table containing a footprint of the low resolution encoder imperfections can be derived and used to adjust the encoder events according their pulse index number.

Experiments show the improvement of the proposed method in practical applications. The position estimation obtained with a second order polynomial fit through five encoder events reduces the position error up to 33% compared to the quantized position signal of the low resolution encoder. If the encoder imperfections are calibrated and the encoder events are adjusted accordingly, the position error can even be reduced by 87%. Moreover the estimated speed is much smoother than the differentiated quantized low resolution encoder signal.

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