Foot placement for balance in planar bipeds with point feet

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1 Introduction

When humanoid robots are going to be used in society, they should be capable to maintain their balance. Knowing where to step appears to be important to remain balanced. Here we present an algorithm to determine proper foot placement (FP) for balance of planar bipeds with point feet [1]. The algorithm is based on conservation of energy, taking into account energy losses at impact.

2 Biped model

We model a biped as a chain of N rigid bodies from stance to swing foot interconnected by revolute joints. The stance and swing leg swap immediately at the impact of the swing foot with the ground, so that the biped can be modeled as a system with impulsive effects:

$$\begin{cases} D(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=u, & q_- \notin S, \\ \dot{q}_+ = \Delta(q_-)\dot{q}_-, & q_- \in S, \end{cases} \tag{1}$$

where $q \in \mathbb{T}^N$ is the state vector, $D \in \mathbb{R}^{N\times N}$ is the inertia matrix, $C\dot{q} \in \mathbb{R}^N$ is the Coriolis and centrifugal vector, $G \in \mathbb{R}^N$ is the gravity vector and $u \in \mathbb{R}^N$ is the input vector. The impact map is given by $\Delta \in \mathbb{R}^{N\times N}$ and S is the impact surface. Subscripts $_-$, $_+$ and $_b$ indicate states just before impact, just after impact and in a balanced configuration respectively.

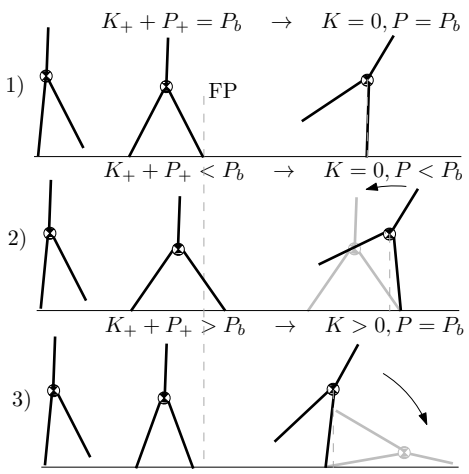

Figure 1: Schematic drawing of the relation between foot placement and balance. The kinetic and potential energy are K and P respectively ($_+$ indicates just after impact). The potential energy in a balanced configuration is P_b.

3 Foot placement and balance

A balanced configuration is a configuration in which the center of mass of the biped lies exactly above its stance foot. A biped is in balance as long as it can still place its swing foot at a location such that it evolves to a balanced configuration, see Figure 1:

1) The biped steps onto the FP point and it stops exactly at the balanced configuration.
2) The biped steps after the FP point, does not reach the balanced configuration and falls backward.
3) The biped steps before the FP point, does not stop at the balanced configuration and falls forward.

The desired FP for balance can now be found if we assume that the sum of the kinetic energy K and the potential energy P is conserved after impact:

$$K(q_+,\dot{q}_+)+P(q_+)=K(q_-,\Delta(q_-)\dot{q}_-)+P(q_-)=P_b. \tag{2}$$

Solving (2) for q_- yields the desired location that indicates where the foot would need to be placed to balance the biped if the impact were to occur in the next instant.

4 Simulation results

We model a three link biped according to (1). At each time step we solve (2) for q_- and use this as reference q_r for a feedback tracking controller. Results of simulations of situation 2) and 3) are shown in Figure 2. The simulations show that the biped is continuously in balance and that it can be controlled to stably walk or stop by proper foot placement.

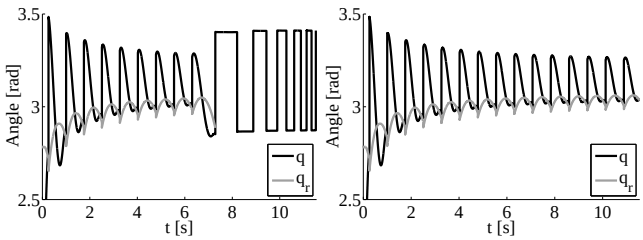

Figure 2: Simulation results: actual configuration (q) and desired one at the end of a step (q_r) for stepping after (left) and before (right) the FPE

References

[1] P. v. Zutven, D. Kostić, and H. Nijmeijer. Foot placement for planar bipeds with point feet. *ICRA*, 2012, in press.