

# Parameter identification of robotic systems with series elastic actuators

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**Abstract:** This paper presents a framework for identification of parameters of a robotic system with series elastic actuation. A model of the series elastic actuation is presented and a method for identification of the model parameters is proposed based on a regressor form of the system model. An algorithm is contributed for automatic derivation of the regressor form from the system equations of motion. The suggested framework is validated in experiments on a humanoid robot equipped with series elastic actuated joints.

Keywords: identification of robot dynamics, series elastic actuation, optimal identification experiments

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## 1. INTRODUCTION

Classically, a stiff design of a drive train between an actuator and a robot joint is recommended. The idea is that a non-stiff drive train limits the controller bandwidth and in that sense decreases accuracy of position control. However, on purpose adding an elastic element with known properties into the drive train can be beneficial. This fairly new concept is known as series elastic actuation. Compared to the classical approach, the series elastic actuation offers less unintentional damage to the environment, the possibility to store energy, better shock tolerance, lower reflected inertia and higher force control accuracy, Pratt and Williamson [1995]. On the other hand, adding an elastic element in the drive train may influence the position tracking performance of the system if a linear feedback design is used. To cope with decreasing control bandwidth, which is inherent to linear feedback design, more advanced, in particular model based feedforward/feedback control architectures are needed, Pratt and Williamson [1995], Vallery et al. [2007]. Performance of model-based control architectures heavily relies on knowledge of the system parameters (geometrical ones, like lengths, and inertial ones, like mass or inertia). Accurate knowledge of these parameters is also vital for analysis and control design of systems with series elastic actuation.

The research field of robot identification has been established decades ago, An et al. [1985], Gautier and Khalil [1989, 1990], but as new robotic systems are introduced, robotic identification techniques are evolving too. Nowadays, series elastic actuated systems increase in popularity, posing new challenges in modeling, analysis, system identification and control design. Unfortunately, there is limited literature available on modeling and identification of robots with series elastic actuation. Hence, in this paper we contribute a systematic method to model and estimate geometric and inertial parameters of such robotic systems. Unlike a common way to identify series elastic actuation using static measurements, in this paper we propose

a dynamic system identification, Calafiore et al. [2001], Kawasaki et al. [1996], using a regressor form of a model of the series elastic actuation. Our method offers a more efficient identification than static measurements, since it estimates parameters of actuator, drive train and load at once with only one set of experimental data. Use of the regressor form of the system equations of motion facilitates dynamic identification, Gautier and Khalil [1989], Calafiore et al. [2001], however, representing the general Euler-Lagrange dynamics into the regressor form is a difficult task, Spong et al. [2006]. In this paper, we resolve this difficulty by means of an algorithm which automatically derives the regressor form for given Euler-Lagrange equations of motion. An important merit of our algorithm is that it computes the base regressor form automatically, which only contains the minimum set of identifiable system parameters. This algorithm is the main contribution of this paper, since it delivers the regressor form for any Euler-Lagrange system, unlike the existing methods that can derive the regressor form only for particular kinematic configurations of robotic systems, Gautier and Khalil [1990]. Another contribution is the experimental validation of our modeling and identification strategy on a humanoid robot TULip, Hobbelen et al. [2008].

The paper is organized as follows. In section 2, a model of a typical series elastic actuation, is described. In section 3, the identification method is explained and an automatic algorithm to find the regressor form is given. In section 4, theoretical results are validated with experimental results. Concluding remarks are given in the last section.

## 2. MODEL OF SERIES ELASTIC ACTUATION

We consider a general model that describes the dynamics of actuators, the Euler-Lagrange equations of motion of a robotic system, and a drive train with elastic elements. A model of a series elastic drive train is schematically shown in figure 1, together with a schematic drawing of a stiff drive train. This model represents a typical design of series

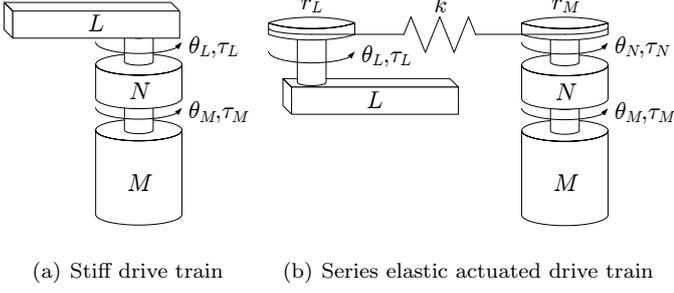


Fig. 1. Schematic drawing of two different drive train concepts

elastic actuators in robotic arms. The model applies to the series elastic actuation of the humanoid robot TULip. A drawing of one leg of this robot with series elastic actuation, is shown in figure 2.

The given model represents an actuated link of a robotic arm and consists of an actuator  $M$ , a load  $L$ , a gearbox  $N$  and two pulleys with radii  $r_M$  and  $r_L$  interconnected by the elastic drive train with stiffness  $k$ . The torque delivered by the motor and torques acting after the gearbox and at the load are denoted by  $\tau_M$ ,  $\tau_N$  and  $\tau_L$ , respectively. The corresponding angular displacements are denoted by  $\theta_M$ ,  $\theta_N$  and  $\theta_L$ , respectively. In the rest of this section, equations of motion of the actuator with gearbox, the drive train, and the load are derived.

### 2.1 Dynamics of an actuator with gearbox

The actuator comprises a geared electric DC motor. This motor is subject to viscous and Coulomb friction. The mechanical equation of motion of the motor in link  $i$  of a robotic arm can be represented by:

$$I_{M,i}\ddot{\theta}_{M,i} + F_{v,i}\dot{\theta}_{M,i} + F_{C,i}\text{sign}(\dot{\theta}_{M,i}) + \frac{\tau_{N,i}}{N_i} = \tau_{M,i}, \quad (1)$$

where  $I_{M,i}$  is the motor shaft inertia and  $F_{v,i}$  and  $F_{C,i}$  are the viscous and Coulomb friction coefficients, respectively. A gearbox is used to increase the output torque from the motor by a factor  $N_i$ , equal to the reduction ratio. If the reduction ratio is high enough, then dynamics of the robotic arm is very much decoupled from the actuator dynamics. Consequently, control of one actuator will hardly be influenced by motions in other joints. Implementation of a gearbox, however, introduces extra friction and causes a decrease in energy efficiency. The latter effect is not taken into account in our model. The angular velocity and acceleration at the output of the gearbox are reduced with respect to the motor side with the same factor  $N_i$ :

$$\begin{aligned} \dot{\theta}_{M,i} &= N_i \dot{\theta}_{N,i}, \\ \ddot{\theta}_{M,i} &= N_i \ddot{\theta}_{N,i}. \end{aligned} \quad (2)$$

In general, the gearbox ratio and the motor shaft inertia are known, unlike the friction parameters that need to be experimentally estimated.

### 2.2 Drive train dynamics

The drive train converts the angular motion of the actuator into a linear displacement using a pulley with radius  $r_{M,i}$ . After the pulley, cables are connected in series with

a spring of stiffness  $k_i$ . The stiffness of the cables can be neglected, if it is orders of magnitude higher than the spring stiffness. At the load side, the linear spring motion is converted to an angular one using a pulley with radius  $r_{L,i}$ . The difference in angular displacements at the actuator and the load sides determines the force exerted by the spring:

$$F_{s,i} = k_i (r_{M,i}\theta_{N,i} - r_{L,i}\theta_{L,i}). \quad (3)$$

This force acts on both sides of the drive train, it induces a load torque for the actuator and actuates the load:

$$\tau_{N,i} = k_i r_{M,i} (r_{M,i}\theta_{N,i} - r_{L,i}\theta_{L,i}), \quad (4a)$$

$$\tau_{L,i} = k_i r_{L,i} (r_{M,i}\theta_{N,i} - r_{L,i}\theta_{L,i}). \quad (4b)$$

In practice, the pulley radii are known, while the spring stiffness needs to be identified.

### 2.3 Load dynamics

The considered system represents a chain of several links that form a robotic arm. Each link is characterized by mass  $m_i$ , a symmetric inertia tensor  $I_i$ , length  $l_i$  and vector  $r_i$  determining position of the link center of mass. Equations of motion of the robotic arm can be derived using the Euler-Lagrange approach, Spong et al. [2006] and are represented in standard form:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau_L, \quad (5)$$

where  $q \in \mathbb{R}^n$  is the vector of joint displacements,  $D(q) \in \mathbb{R}^{n \times n}$  is the inertia matrix,  $C(q,\dot{q}) \in \mathbb{R}^{n \times n}$  is the matrix appearing in the term representing Coriolis and centrifugal effects,  $G(q) \in \mathbb{R}^n$  is the gravity vector and  $n$  is the number of robot links. To be consistent with the equations of the actuator dynamics and the drive train, by defining  $q = [\theta_{L,1} \cdots \theta_{L,n}]^T$  we can rewrite (5) for each link  $i$  separately:

$$\sum_{j=1}^n D_{ij}\ddot{\theta}_{L,j} + \sum_{j=1}^n C_{ij}\dot{\theta}_{L,j} + G_i = \tau_{L,i}. \quad (6)$$

In general, the lengths of the links are directly measurable, whereas the mass, inertia and position of the center of mass are unknown and need to be estimated.

### 2.4 Complete model

The complete model combines dynamics of the actuator, drive train and load. We present the model in two parts, one for the actuator side and another for the load side, because this is more suitable for system identification. In the case of the series elastic actuation, one can control the torque applied by the motor  $\tau_{M,i}$ , which is the input to the actuator part of the model. If the angular position of the motor shaft  $\theta_{M,i}$  and the angular position of the load  $\theta_{L,i}$  are directly measurable, e.g. using incremental encoders, then these variables represent the outputs of the full model. Combining (1), (2) and (4a) the part of the model at the actuator side can be written as:

$$I_{M,i}\ddot{\theta}_{M,i} + F_{v,i}\dot{\theta}_{M,i} + F_{C,i}\text{sign}(\dot{\theta}_{M,i}) + \frac{k_i r_{M,i}}{N_i} \left( \frac{r_{M,i}}{N_i} \theta_{M,i} - r_{L,i} \theta_{L,i} \right) = \tau_{M,i}. \quad (7)$$

Unfortunately, we cannot directly apply the torque at the load side  $\tau_{L,i}$ , but only indirectly via the spring. Therefore, the input to the part of the model at the load side has to

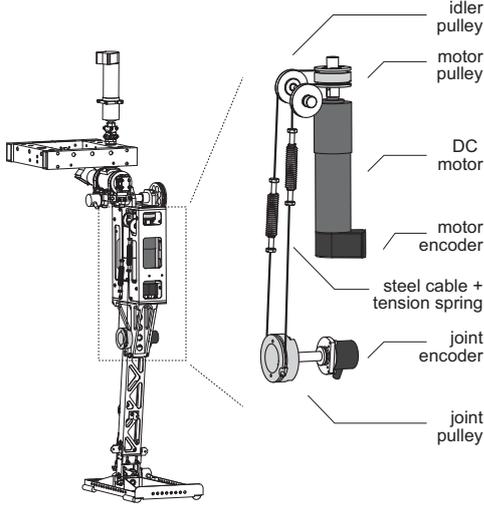


Fig. 2. Drawing of series elastic actuation in humanoid robot TULIP

be calculated from (4b). By combining (4b) with (2) and (6), the part of the model at the load side is derived:

$$k_i r_{L,i} \left( \frac{r_{M,i}}{N_i} \theta_{M,i} - r_{L,i} \theta_{L,i} \right) = \sum_{j=1}^n D_{ij} \ddot{\theta}_{L,j} + \sum_{j=1}^n C_{ij} \dot{\theta}_{L,j} + G_i. \quad (8)$$

This complete model (7) and (8) is the starting point for identification of the unknown system parameters. The complete model is also suitable for system analysis and control design.

### 3. IDENTIFICATION ALGORITHM

The parameters of the model need to be identified in order to describe the physical behavior of the system as close as possible. The model parameters can be separated in two groups: geometrical and inertial ones. Some parameters can be measured directly using static identification experiments; others need to be estimated in dynamic experiments. Dynamic experiments become inevitable especially when a robotic arm is already assembled, since on an assembled arm it is difficult to achieve static measurements of parameters of individual links.

To facilitate estimation of the model parameters, the system model should be written in the so-called regressor form, Gautier and Khalil [1988], Kawasaki et al. [1996], Calafiore et al. [2001], Waiboer et al. [2005]:

$$\zeta = \mathbf{R}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \boldsymbol{\vartheta}, \quad (9)$$

where  $\zeta \in \mathbb{R}^n$  is a vector of parameter independent expressions,  $\mathbf{R}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in \mathbb{R}^{n \times m}$  is the regressor matrix,  $\boldsymbol{\vartheta} \in \mathbb{R}^m$  is a vector of unknown parameters,  $n$  is the number of the robot links, and  $m$  is the number of unknown parameters. For reliable system identification, the columns of the regressor matrix must be linearly independent, which is the so-called base regressor form, Gautier and Khalil [1989, 1990], Kawasaki et al. [1996]. In the following, subscript 0 will be used to denote the base regressor form. In particular,  $\mathbf{R}_0(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in \mathbb{R}^{n \times p}$  will be the base regressor matrix,  $\boldsymbol{\vartheta}_0 \in \mathbb{R}^p$  the base parameter vector, and  $p$  the number of base parameters. Finding the

base regressor form is not a trivial task, since the Euler-Lagrange equations of motion may contain a large number of trigonometric terms and nonlinear couplings between the system coordinates.

In the rest of this section an algorithm will be proposed that automatically finds the base regressor form starting from the complete system model presented in section 2.4.

#### 3.1 Derivation of the regressor form

By assuming that the moment of inertia  $I_{M,i}$  of the motor shaft is known, it is easy to find the regressor form of part of the system model at the actuator side:

$$\begin{aligned} \zeta_i &= \tau_{M,i} - I_{M,i} \ddot{\theta}_{M,i}, \\ \mathbf{R}_i &= \begin{bmatrix} \dot{\theta}_{M,i} \\ \text{sign}(\dot{\theta}_{M,i}) \\ \frac{r_{M,i}}{N_i} \left( \frac{r_{M,i}}{N_i} \theta_{M,i} - r_{L,i} \theta_{L,i} \right) \end{bmatrix}^T, \quad (10) \\ \boldsymbol{\vartheta}_i &= \begin{bmatrix} B_{M,i} \\ F_{c,i} \\ k_i \end{bmatrix}. \end{aligned}$$

Unfortunately, nonlinearity and complexity of Euler-Lagrange equations of motions make finding the regressor form of the system model at the load side a difficult task. That is why we develop an algorithm that can automatically separate mutually different expressions from the equations of motion that depend on the system states and their time-derivatives. After separation, this algorithm puts the expressions into the appropriate matrix. The algorithm utilizes the fact that one can convert the equations of motion to text based strings. Within these strings, it is straightforward to locate mathematical operators, such as +, -, \* and /. These operators, in fact, divide the given equations in different expressions. Each expression belongs to a certain group: a parameter independent expression, expression that makes a product with unknown parameter in the equation of motion, or unknown parameter. In this way, we can work ourselves through all equations and put every expression in the corresponding matrix according to the regressor model (9): parameter independent expressions constitute the vector  $\zeta$ , expressions that make products with unknown parameters become elements of the matrix  $\mathbf{R}$ , and unknown parameters make the vector  $\boldsymbol{\vartheta}$ . Consequently, we build up the regressor form (9) in an automated way, as described by algorithm 1.

The algorithm turns out to be fast and can be applied to different systems. It is successfully tested during identification of a Philips robotic arm containing seven degrees of freedom, Philips Applied Technologies [2009].

#### 3.2 Derivation of the base regressor form

The base parameter set is a minimal set of identifiable parameters that describes the system completely, Gautier and Khalil [1989, 1990]. This set corresponds to the regressor matrix  $\mathbf{R}$  with mutually linearly independent columns. As it can be seen from (9), the regressor form of the part of the system model at the actuator side (10) is already in the base regressor form. On the other hand, algorithm 1 does not deliver the base regressor form in the general case. To facilitate automatic conversion

**Algorithm 1.** Regression algorithm

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for  $r \leftarrow 1, n$  do
   $e \leftarrow \text{str}(eom(r))$ 
   $pmop \leftarrow \text{sort}([\text{strfind}(e, ' + '), \text{strfind}(e, ' - ')])$ 
  for  $i \leftarrow 1, \text{len}(pmop) - 1$  do
     $pms \leftarrow e(pmop(i) + 1 : pmop(i + 1) - 1)$ 
     $mdop \leftarrow [\text{strfind}(pms, '*'), \text{strfind}(pms, '/')]$ 
    for  $j \leftarrow 1, \text{len}(mdop) - 1$  do
       $m ds \leftarrow pms(mdop(j) + 1 : mdop(j + 1) - 1)$ 
      if  $m ds$  is dependent expression then
         $R(r, :) \leftarrow [R(r, :), m ds]$ 
      else if  $m ds$  is independent expression then
         $\zeta \leftarrow [\zeta; m ds]$ 
      else
         $\vartheta \leftarrow [\vartheta; m ds]$ 
      end if
    end for
  end for
end for

```

of the regressor form (9) into the base regressor form, the reasoning proposed in Kawasaki et al. [1996] can be used. The regressor matrix  $\mathbf{R}$  contains expressions that are time-varying functions of joint variables and their time-derivatives. Some of these expressions also contain known system parameters (mostly geometric once). A function of joint variables that appears in one or several expressions of the regressor is called a fundamental function. The fundamental functions are stored in a vector  $\mathbf{f} \in \mathbb{R}^k$ , where  $k$  is the number of fundamental functions in  $\mathbf{R}$ , whereas the known system parameters are stored in a vector  $\mathbf{b}_{ij} \in \mathbb{R}^k$  such that the following holds:

$$R_{ij}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \mathbf{f}^T(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \mathbf{b}_{ij}, \quad (11)$$

Here,  $i$  and  $j$  represent a row and column index in the regressor matrix  $\mathbf{R}$ , so for each element  $R_{ij}$  in  $\mathbf{R}$  one can find the corresponding vector  $\mathbf{b}_{ij}$ . All vectors  $\mathbf{b}_{ij}$  can be stored in one matrix  $\mathbf{B} \in \mathbb{R}^{l \times m}$ ,  $l = nk$ , as follows:

$$\mathbf{B} = \begin{bmatrix} \mathbf{b}_{11} & \cdots & \mathbf{b}_{1m} \\ \vdots & \ddots & \vdots \\ \mathbf{b}_{l1} & \cdots & \mathbf{b}_{lm} \end{bmatrix}. \quad (12)$$

This matrix  $\mathbf{B}$  can be transformed to the Echelon form  $\mathbf{B}_E$  by performing a Gauss-Jordan elimination, Kawasaki et al. [1996]. The result is an upper triangular matrix of the following form:

$$\mathbf{B}_E = \begin{bmatrix} \mathbf{B}_{Eu} \\ \mathbf{0} \end{bmatrix}. \quad (13)$$

The matrix  $\mathbf{B}_E$  consists of columns that can be characterized according to three types. The first type has all elements equal to zero. This means that the corresponding column in the regressor  $\mathbf{R}$  is not needed for parameter identification and can be removed. The second type is a column in which all elements are zero except from only one element which is equal to 1. This means that the corresponding column in regressor  $\mathbf{R}$  is necessary and linearly independent from the other columns. The last type is a column with zeros and known parameter elements. The corresponding column in regressor  $\mathbf{R}$  is linearly dependent on other columns and can be removed. The corresponding base parameter vector  $\vartheta_0$  can be derived in the following way:

$$\vartheta_0 = \mathbf{B}_{Eu} \vartheta. \quad (14)$$

This completes derivation of the minimal regressor form (9) which contains the base parameter set.

## 4. EXPERIMENTAL VALIDATION

The system identification approach presented in the previous sections is validated in experiments. Since in general, the number of the base parameters is not equal to number of equations of motion ( $p \neq n$ ), the base parameters cannot be estimated uniquely. Instead, we determine them by means of some algorithm for optimal fitting. Reliability of the parameter fit can be improved by collecting system inputs and outputs at more time instants. In other words, by letting the system move along trajectories consisting of several data points. With  $r$  denoting the number of data points, we create the following compact regressor form:

$$\mathbf{y} = \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \vartheta_0, \quad (15)$$

$$\text{where } \mathbf{y} = \begin{bmatrix} \zeta(t_1) \\ \vdots \\ \zeta(t_r) \end{bmatrix} \text{ and } \mathbf{H} = \begin{bmatrix} \mathbf{R}_0(\mathbf{q}(t_1), \dot{\mathbf{q}}(t_1), \ddot{\mathbf{q}}(t_1)) \\ \vdots \\ \mathbf{R}_0(\mathbf{q}(t_r), \dot{\mathbf{q}}(t_r), \ddot{\mathbf{q}}(t_r)) \end{bmatrix}.$$

In this way all data points are incorporated in the estimation process and typically more equations than the number of base parameters are obtained ( $rn > p$ ). The base parameters are estimated by fitting the model to the experimental data using a least squares or maximum likelihood algorithm, Calafiore et al. [2001], Swevers et al. [1997], Olsen and Petersen [2001]. As it can be seen from (15), the regressor and therefore also quality of the estimation depend on motion of the system during the identification experiment. For reliable parameter estimation, all relevant system dynamics should be sufficiently excited. System trajectories that satisfy such a property are called persistently exciting, Slotine and Li [1991].

### 4.1 Persistently exciting trajectories

Besides persistently exciting for system dynamics, optimal system trajectories in an identification experiment should also satisfy mechanical limits in the robot joints. Moreover, actuation capabilities should also be taken into account by limiting velocities and accelerations in the joints. A usual way to find a persistently exciting trajectory is to minimize the  $l_2$ -norm condition number of the regressor matrix, Calafiore et al. [2001], Swevers et al. [1997], Gautier and Khalil [1991]:

$$\kappa(\mathbf{H}) = \left| \frac{\lambda_{max}(\mathbf{H})}{\lambda_{min}(\mathbf{H})} \right|, \quad (16)$$

where  $\lambda_{max}(\mathbf{H})$  and  $\lambda_{min}(\mathbf{H})$  are the maximum and minimum eigenvalues of  $\mathbf{H}$ , respectively. By taking into account the constraints on the trajectories, we consider the following optimization problem:

$$\min_{\mathbf{q}_r} \kappa(\mathbf{H}(\mathbf{q}_r, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)), \quad \begin{aligned} \mathbf{c}_{lp} &\leq \mathbf{q}_r \leq \mathbf{c}_{up}, \\ \mathbf{c}_{lv} &\leq \dot{\mathbf{q}}_r \leq \mathbf{c}_{uv}, \\ \mathbf{c}_{la} &\leq \ddot{\mathbf{q}}_r \leq \mathbf{c}_{ua}, \end{aligned} \quad (17)$$

where  $\mathbf{q}_r$  are trajectories in the robot joints,  $\dot{\mathbf{q}}_r$  and  $\ddot{\mathbf{q}}_r$  are time-derivatives of these trajectories,  $\mathbf{c}_{lp}$ ,  $\mathbf{c}_{lv}$  and  $\mathbf{c}_{la}$  are the lower boundary constraints on position, velocity and acceleration, respectively, and  $\mathbf{c}_{up}$ ,  $\mathbf{c}_{uv}$  and  $\mathbf{c}_{ua}$  are the corresponding upper boundary constraints. As one can see, this is a constrained nonlinear optimization problem and there exist several optimization algorithms to solve

this problem in a local fashion. Solutions usually depend on the initial conditions, since the considered optimization problem is not convex. Finding joint trajectories that optimize (17) can be very time consuming for systems with a large number of links. Therefore, it is more pragmatic to postulate the trajectory in a form of a finitely parameterized function, such as a finite Fourier series or polynomial, Kostić et al. [2004]. For instance, consider as arguments for optimization, the coefficients  $\mathbf{a}_i$ ,  $i = 0 \dots d$  of the polynomials of order  $d$ :

$$\mathbf{q}_r(t) = \sum_{i=0}^d a_i t^i. \quad (18)$$

The number of optimization arguments can be kept low to speed-up the optimization process. The corresponding optimization problem becomes:

$$\min_{\mathbf{a}_i} \kappa(\mathbf{H}(\mathbf{q}_r, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)), \quad \begin{aligned} \mathbf{c}_{lp} &\leq \mathbf{q}_r \leq \mathbf{c}_{up}, \\ \mathbf{c}_{lv} &\leq \dot{\mathbf{q}}_r \leq \mathbf{c}_{uv}, \\ \mathbf{c}_{la} &\leq \ddot{\mathbf{q}}_r \leq \mathbf{c}_{ua}, \end{aligned} \quad (19)$$

In our experimental case-study, parameterization using polynomial functions turns out to be successful, time-efficient, and fairly easy to implement. We design the optimal polynomial trajectories of a duration of 10 s. In experiments, we realize these trajectories with a sampling frequency of 1 kHz.

#### 4.2 Experimental set-up: humanoid robot TULip

The persistently exciting trajectories designed in the previous section are used to estimate the model parameters of one leg of the humanoid robot TULip, Hobbelen et al. [2008]. This leg consists of five interconnected links, from which three are actuated using series elastic actuators. A drawing of one leg of the robot is shown in figure 2. A DC motor actuates the corresponding joint through a double pulley construction with two linear springs placed in series. The rotation of the motor shaft can be measured with an incremental motor encoder, and the rotation of the joint with an incremental joint encoder. The torque applied by the motor is the input, and the both measured rotations are the outputs of the system.

#### 4.3 Experimental results

An experiment with persistently exciting trajectories has been carried out. Because we build up the model in two parts, we are able to separately estimate the actuator, drive train and load parameters in one experiment. Firstly, we estimate the viscous and Coulomb friction in the motor and the stiffness of the spring in the drive train, by filling in the regressor of part of the model at the actuator side (10) for every time step in (15). The unknown parameters are estimated using a least squares algorithm, Olsen and Petersen [2001], and shown in table 1.

To evaluate quality of this estimation, we can compute torques for given joint motions using the model with the estimated parameters filled in and compare these computed torques with the torques measured as the system performs the same motions. By visual inspection, the best and worst result of such an evaluation experiment are depicted in figure 3.

As can be seen, the model-based estimated torque has

Table 1. Estimated actuator and drive train parameters

Parameter	Estimated value	Variance
$B_{m3}$	0.00008	0.0000045
$F_{c3}$	0.00456	0.0000745
$k_3$	83008	131
$B_{m4}$	0.00008	0.0000021
$F_{c4}$	0.00268	0.0001168
$k_4$	59966	90
$B_{m5}$	0.00004	0.0000227
$F_{c5}$	0.01178	0.0004170
$k_5$	51394	801

a high overlap with the measured one. The chattering behavior in both figures is caused by the Coulomb friction and indicates that the friction model is not accurate enough. The misalignment of the two graphs in figure 3(b) is caused by higher order dynamics that is not taken into account in the model.

The next step is to estimate unknown parameters of part of the model at the load side. Since we cannot directly measure the torque applied to the load, we have to estimate it using (4b). We derive the regressor of part of the model at the load side from (8) using the automatic algorithm 1 and estimate the load parameters in a way which is equivalent to our approach at the actuator side. For validation of estimates of the load parameters, we can show figure 4.

Figure 4 reveals that similar errors as on the actuator side appear, mostly caused by an inaccurate friction model and unmodeled higher order dynamics. Despite these errors, we can conclude that our approach to parameter estimation is successfully validated on a realistic system with series elastic actuation.

## 5. CONCLUSION

We offer an approach to identify dynamics of robotic systems with series elastic actuation. A framework to model such robotic systems is proposed, which covers dynamics of the robot actuators, drive trains and nonlinear rigid-body dynamics. The main contribution is an algorithm for automatic conversion of the system model into a regressor form of minimum order. This algorithm can be applied to general Euler-Lagrange systems and not only to particular kinematic configurations, as the existing methods. We validate our theoretical results in experiments on a realistic humanoid robot. This experimental validation gives system identification results of sufficiently good quality.

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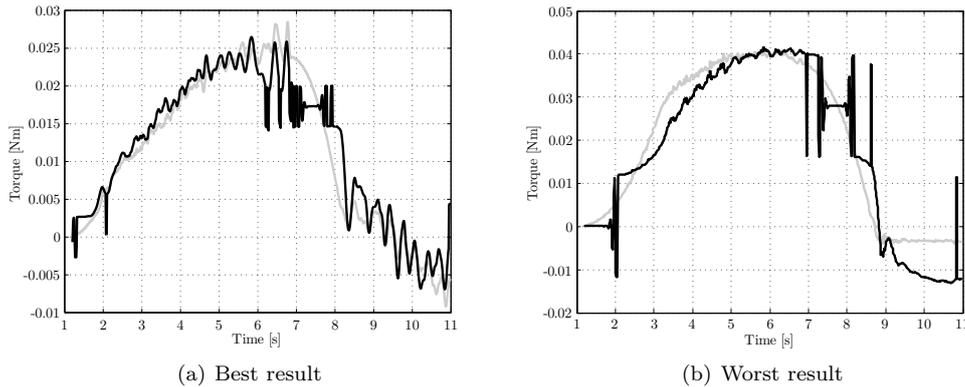


Fig. 3. Experimental validation results for actuator side: measured torque (grey) and calculated torque (black)

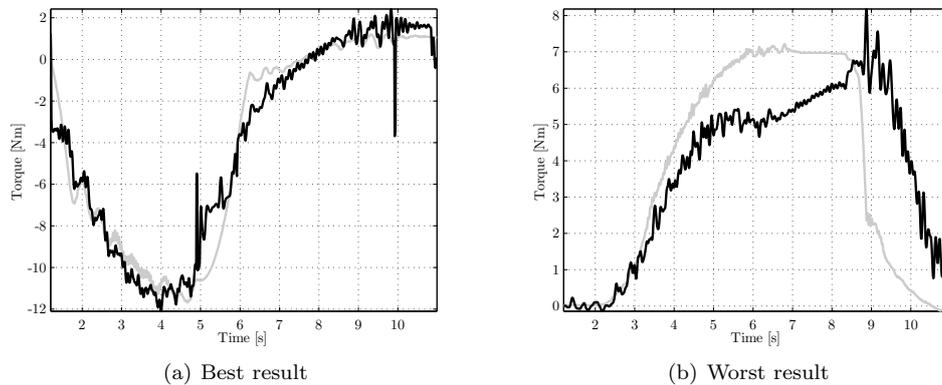


Fig. 4. Experimental validation results for load side: measured torque (grey) and calculated torque (black)

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